Calculus (Functions, Derivatives & Integration) — Top 30 PYQs

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1. If the function $f(x) = \frac{\sin 3x + \alpha \sin x - \beta \cos 3x}{x^3}$, $x \in \mathbb{R}$, is continuous at $x = 0$, then $f(0)$ is equal to :

(A) 2 **(B)** -2

(C) 4 **(D)** -4

Differentiation of f(x) with respect to g(x) · JEE Main 2020 · Shift-II — View Solution on Crucible ↗

2. The derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at $x = \frac{1}{2}$ is :

(A) $\frac{2\sqrt{3}}{5}$ **(B)** $\frac{\sqrt{3}}{12}$

(C) $\frac{2\sqrt{3}}{3}$ **(D)** $\frac{\sqrt{3}}{10}$

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3. The sum of the absolute maximum and absolute minimum values of the function $f(x) = \tan^{-1}(\sin x - \cos x)$ in the interval $[0, \pi]$ is

(A) 0 **(B)** $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right) - \frac{\pi}{4}$

(C) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right) - \frac{\pi}{4}$ **(D)** $\frac{-\pi}{12}$

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4. Let $f(x) = \int \frac{2x}{(x^2+1)(x^2+3)}\,dx$. If $f(3) = \frac{1}{2}(\log_e 5 - \log_e 6)$, then $f(4)$ is equal to

(A) $\frac{1}{2}(\log_e 17 - \log_e 19)$ **(B)** $\log_e 17 - \log_e 18$

(C) $\frac{1}{2}(\log_e 19 - \log_e 17)$ **(D)** $\log_e 19 - \log_e 20$

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5. The value of $\int_0^1 (2x^3 - 3x^2 - x + 1)^{\frac{1}{3}}\,dx$ is equal to:

(A) 0 **(B)** 1

(C) 2 **(D)** -1

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6. The area (in sq. units) of the region bounded by the parabola, $y = x^2 + 2$ and the lines, $y = x + 1, x = 0$ and $x = 3$, is

(A) $\frac{17}{4}$ **(B)** $\frac{21}{2}$

(C) $\frac{15}{2}$ **(D)** $\frac{15}{4}$

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7. The solution curve, of the differential equation $2y\frac{dy}{dx} + 3 = 5\frac{dy}{dx}$, passing through the point $(0, 1)$ is a conic, whose vertex lies on the line:

(A) $2x + 3y = 9$ **(B)** $2x + 3y = -9$

(C) $2x + 3y = -6$ **(D)** $2x + 3y = 6$

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8. The function $f : \mathbb{R} \to \mathbb{R}, f(x) = \frac{x^2+2x-15}{x^2-4x+9}, x \in \mathbb{R}$ is

(A) one-one but not onto. **(B)** both one-one and onto.

(C) onto but not one-one. **(D)** neither one-one nor onto.

Limit using Trigonometry & Algebraic forms · JEE Main 2025 · Shift-I View Solution on Crucible ↗

9. $\lim_{x\to 0} \operatorname{cosec} x \left(\sqrt{2\cos^2 x + 3\cos x} - \sqrt{\cos^2 x + \sin x + 4}\right)$ is:

(A) 0 **(B)** $\frac{1}{\sqrt{15}}$

(C) $\frac{1}{2\sqrt{5}}$ **(D)** $-\frac{1}{2\sqrt{5}}$

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10. Let $f : [-1, 2] \to \mathbb{R}$ be given by $f(x) = 2x^2 + x + [x^2] - [x]$, where $[t]$ denotes the greatest integer less than or equal to t. The number of points, where f is not continuous, is :

(A) 5 **(B)** 6

(C) 3 **(D)** 4

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11. If $y(\theta)=\frac{2\cos\theta+\cos 2\theta}{\cos 3\theta+4\cos 2\theta+5\cos\theta+2}$, then at $\theta=\frac{\pi}{2}$, $y''+y'+y$ is equal to :

(A) $\frac{1}{2}$ **(B)** 1

(C) 2 **(D)** $\frac{3}{2}$

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12. The local maximum of the slope of the curve $y=\frac{1}{2}x^4-5x^3+18x^2-19x$ occurs at the point

(A) $\left(3,\frac{21}{2}\right)$ **(B)** $(2,2)$

(C) $(2,9)$ **(D)** $(0,0)$

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13. The integral $\int\frac{2x^3-1}{x^4+x}\,dx$, is equal to

(A) $\frac{1}{2}\log_e\frac{(x^3+1)^2}{|x^3|}+C$ **(B)** $\log_e\frac{|x^3+1|}{x^2}+C$

(C) $\log_e\left|\frac{(x^3+1)}{x}\right|+C$ **(D)** $\frac{1}{2}\log_e\frac{|x^3+1|}{x^2}+C$

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14. For $0<a<1$, the value of the integral $\int_0^{\pi}\frac{dx}{1-2a\cos x+a^2}$ is :

(A) $\frac{\pi^2}{\pi+a^2}$ **(B)** $\frac{\pi^2}{\pi-a^2}$

(C) $\frac{\pi}{1-a^2}$ **(D)** $\frac{\pi}{1+a^2}$

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15. If the area of the region $\{(x,y):1+x^2\le y\le\min\{x+7,11-3x\}\}$ is A, then $3A$ is equal to

(A) 50 **(B)** 49

(C) 46 **(D)** 47

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16. Let $y = y(x)$ be a solution of the differential equation, $\sqrt{1-x^2}\frac{dy}{dx} + \sqrt{1-y^2} = 0, |x| < 1$. If $y\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$, then $y\left(\frac{-1}{\sqrt{2}}\right)$ is equal to

(A) $\frac{\sqrt{3}}{2}$
(B) $-\frac{1}{\sqrt{2}}$
(C) $\frac{1}{\sqrt{2}}$
(D) $-\frac{\sqrt{3}}{2}$

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17. Let $f(x) = 2x^n + \lambda, \lambda \in \mathbb{R}, n \in \mathbb{N}$, and $f(4) = 133, f(5) = 255$. Then the sum of all the positive integer divisors of $(f(3) - f(2))$ is

(A) 61
(B) 60
(C) 58
(D) 59

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18. $\lim_{x\to 0} \frac{e^{2|\sin x|} - 2|\sin x| - 1}{x^2}$

(A) is equal to -1
(B) does not exist
(C) is equal to 1
(D) is equal to 2

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19. Let $f : R - \{0\} \to R$ be a function satisfying $f\left(\frac{x}{y}\right) = \frac{f(x)}{f(y)}$ for all x, y, $f(y) \neq 0$. If $f'(1) = 2024$, then

(A) $xf'(x) - 2024f(x) = 0$
(B) $xf'(x) + 2024f(x) = 0$
(C) $xf'(x) + f(x) = 2024$
(D) $xf'(x) - 2023f(x) = 0$

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20. If $(a + \sqrt{2}b\cos x)(a - \sqrt{2}b\cos y) = a^2 - b^2$, where $a > b > 0$, then $\frac{dx}{dy}$ at $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$ is:

(A) $\frac{a-2b}{a+2b}$
(B) $\frac{a-b}{a+b}$
(C) $\frac{a+b}{a-b}$
(D) $\frac{2a+b}{2a-b}$

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21. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$, where $a > 0$, attains its local maximum and local minimum values at p and q, respectively, such that $p^2 = q$, then $f(3)$ is equal to:

(A) 55 **(B)** 10

(C) 23 **(D)** 37

Integration by parts: e^x·[f(x) + f'(x)] · JEE Main 2025 · Shift-II View Solution on Crucible ↗

22. If $\int e^x \left(\frac{x\sin^{-1}x}{\sqrt{1-x^2}} + \frac{\sin^{-1}x}{(1-x^2)^{3/2}} + \frac{x}{1-x^2}\right) dx = g(x) + C$, where C is the constant of integration, then $g\left(\frac{1}{2}\right)$ equals :

(A) $\frac{\pi}{4}\sqrt{\frac{e}{3}}$ **(B)** $\frac{\pi}{6}\sqrt{\frac{e}{3}}$

(C) $\frac{\pi}{4}\sqrt{\frac{e}{2}}$ **(D)** $\frac{\pi}{6}\sqrt{\frac{e}{2}}$

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23. The value of the integral $\int_0^1 \frac{\sqrt{x}\,dx}{(1+x)(1+3x)(3+x)}$ is:

(A) $\frac{\pi}{4}\left(1 - \frac{\sqrt{3}}{2}\right)$ **(B)** $\frac{\pi}{8}\left(1 - \frac{\sqrt{3}}{6}\right)$

(C) $\frac{\pi}{8}\left(1 - \frac{\sqrt{3}}{2}\right)$ **(D)** $\frac{\pi}{4}\left(1 - \frac{\sqrt{3}}{6}\right)$

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24. Consider a region $R = \{(x, y) \in R^2 : x^2 \le y \le 2x\}$. If a line $y = \alpha$ divides the area of region R into two equal parts, then which of the following is true ?

(A) $\alpha^3 - 6\alpha^2 + 16 = 0$ **(B)** $3\alpha^2 - 8\alpha^{3/2} + 8 = 0$

(C) $3\alpha^2 - 8\alpha + 8 = 0$ **(D)** $\alpha^3 - 6\alpha^{3/2} - 16 = 0$

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25. The population $P = P(t)$ at time t of a certain species follows the differential equation $\frac{dP}{dt} = 0.5P - 450$. If $P(0) = 850$, then the time at which population becomes zero is:

(A) $\log_e 9$ **(B)** $2\log_e 18$

(C) $\log_e 18$ **(D)** $\frac{1}{2}\log_e 18$

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26. If the domain of the function $f(x) = \sin^{-1}\left(\frac{x-1}{2x+3}\right)$ is $\mathbb{R} - (\alpha, \beta)$, then $12\alpha\beta$ is equal to :

(A) 32
(B) 40
(C) 24
(D) 36

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27. The value of $\lim_{x\to 0^+} \frac{\cos^{-1}(x-[x]^2)\cdot\sin^{-1}(x-[x]^2)}{x-x^3}$, where $[x]$ denotes the greatest integer $\leq x$ is:

(A) π
(B) 0
(C) $\frac{\pi}{4}$
(D) $\frac{\pi}{2}$

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28. Let $f(x) = |2x^2 + 5|x| - 3|, x \in R$. If m and n denote the number of points where f is not continuous and not differentiable respectively, then $m + n$ is equal to:

(A) 5
(B) 2
(C) 0
(D) 3

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29. Suppose $f(x) = \frac{(2^x+2^{-x})\tan x\sqrt{\tan^{-1}(x^2-x+1)}}{(7x^2+3x+1)^3}$. Then the value of $f'(0)$ is equal to

(A) π
(B) 0
(C) $\sqrt{\pi}$
(D) $\frac{\pi}{2}$

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30. The area (in sq. units) of the largest rectangle $ABCD$ whose vertices A and B lie on the x-axis and vertices C and D lie on the parabola, $y = x^2 - 1$ below the x-axis, is :

(A) $\frac{2}{3\sqrt{3}}$
(B) $\frac{1}{3\sqrt{3}}$
(C) $\frac{4}{3}$
(D) $\frac{4}{3\sqrt{3}}$

Answer Key (Q1 to Q30)

Click any question number below to open its exact step-by-step solution on The Crucible ↗

1: (D)	2: (D)	3: (C)	4: (A)	5: (A)	6: (C)	7: (A)	8: (D)	9: (D)	10: (D)
11: (C)	12: (B)	13: (C)	14: (C)	15: (A)	16: (C)	17: (B)	18: (D)	19: (A)	20: (C)
21: (D)	22: (B)	23: (C)	24: (B)	25: (B)	26: (A)	27: (D)	28: (D)	29: (C)	30: (D)

Sample Solution — Question 1 (Correct Option: D)

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Q1. If the function $f(x) = \frac{\sin 3x + \alpha \sin x - \beta \cos 3x}{x^3}$, $x \in \mathbb{R}$, is continuous at $x = 0$, then $f(0)$ is equal to :

The denominator is x^3. For a finite limit the numerator must vanish to order x^3, so its constant, x and x^2 coefficients must all be zero. Expand and match coefficients.

Put $x = 0$ in the numerator: $\sin 0 + \alpha \sin 0 - \beta \cos 0 = -\beta$.

So $\beta = 0$.

Now expand the two sine terms.

$\sin 3x = 3x - \frac{(3x)^3}{6} + O(x^5) = 3x - \frac{9x^3}{2} + O(x^5)$

$\sin x = x - \frac{x^3}{6} + O(x^5)$

Numerator $= (3 + \alpha)x + \left(-\frac{9}{2} - \frac{\alpha}{6}\right) x^3 + O(x^5)$

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