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Mixed properties of triangle · JEE Main 2025 · Shift-I

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1. Let the area of a ΔPQR with vertices $P(5,4), Q(-2,4)$ and $R(a,b)$ be 35square units. If its orthocenter and centroid are $O(2,\frac{14}{5})$ and $C(c,d)$ respectively, then $c+2d$ is equal to

(A) $8/3$

(B) $7/3$

(C) 2

(D) 3

JEE Main 2022 · Shift-II

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2. Let A and B be any two 3×3 symmetric and skew symmetric matrices respectively. Then which of the following is **NOT** true?

(A) A^4-B^4 is a symmetric matrix

(B) $AB-BA$ is a symmetric matrix

(C) B^5-A^5 is a skew-symmetric matrix

(D) $AB+BA$ is a skew-symmetric matrix

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3. Let S denote the set of all real values of λ such that the system of equations $\lambda x+y+z=1,\ x+\lambda y+z=1,\ x+y+\lambda z=1$ is inconsistent, then $\sum_{\lambda\in S}\left(|\lambda|^2+|\lambda|\right)$ is equal to

(A) 2

(B) 12

(C) 4

(D) 6

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4. A square is inscribed in the circle $x^2+y^2-10x-6y+30=0$. One side of this square is parallel to $y=x+3$. If (x_i,y_i) are the vertices of the square, then $\Sigma(x_i^2+y_i^2)$ is equal to:

(A) 148

(B) 152

(C) 160

(D) 156

Intersection of parabola and other curves · JEE Main 2025 · Shift-I

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5. Two parabolas have the same focus $(4,3)$ and their directrices are the x-axis and the y-axis, respectively. If these parabolas intersects at the points A and B, then $(AB)^2$ is equal to :

(A) 392

(B) 384

(C) 192

(D) 96

Finding missing parameters · JEE Main 2022 · Shift-II View Solution on Crucible ↗

6. If the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets the line $\frac{x}{7}+\frac{y}{2\sqrt{6}}=1$ on the x-axis and the line $\frac{x}{7}-\frac{y}{2\sqrt{6}}=1$ on the y-axis, then the eccentricity of the ellipse is

(A) $\frac{5}{7}$ **(B)** $\frac{2\sqrt{6}}{7}$

(C) $\frac{3}{7}$ **(D)** $\frac{2\sqrt{5}}{7}$

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7. If $\frac{\pi}{2}\le x\le\frac{3\pi}{4}$, then $\cos^{-1}\left(\frac{12}{13}\cos x+\frac{5}{13}\sin x\right)$ is equal to

(A) $x-\tan^{-1}\frac{4}{3}$ **(B)** $x+\tan^{-1}\frac{4}{5}$

(C) $x-\tan^{-1}\frac{5}{12}$ **(D)** $x+\tan^{-1}\frac{5}{12}$

Equation & Standard forms · JEE Main 2019 · Shift-II View Solution on Crucible ↗

8. If $5x+9=0$ is the directrix of the hyperbola $16x^2-9y^2=144$, then its corresponding focus is:

(A) $(-5,0)$ **(B)** $(5,0)$

(C) $\left(-\frac{5}{3},0\right)$ **(D)** $\left(\frac{5}{3},0\right)$

Solving Equations by Reduction · JEE Main 2024 · Shift-II View Solution on Crucible ↗

9. The number of solutions of the equation $4\sin^2 x-4\cos^3 x+9-4\cos x=0; x\in[-2\pi,2\pi]$ is:

(A) 1 **(B)** 3

(C) 2 **(D)** 0

Questions on Finding Principal Value · JEE Main 2023 · Shift-I View Solution on Crucible ↗

10. $\tan^{-1}\left(\frac{1+\sqrt{3}}{3+\sqrt{3}}\right)+\sec^{-1}\sqrt{\frac{8+4\sqrt{3}}{6+3\sqrt{3}}}$ is equal to

(A) $\frac{\pi}{4}$ **(B)** $\frac{\pi}{2}$

(C) $\frac{\pi}{3}$ **(D)** $\frac{\pi}{6}$

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11. If the four points, whose position vectors are $3\hat{i}-4\hat{j}+2\hat{k}, \hat{i}+2\hat{j}-\hat{k}, -2\hat{i}-\hat{j}+3\hat{k}$ and $5\hat{i}-2\alpha\hat{j}+4\hat{k}$ are coplanar, then α is equal to

(A) $\frac{73}{17}$
(B) $-\frac{107}{17}$
(C) $-\frac{73}{17}$
(D) $\frac{107}{17}$

Line - Intersection of Two Lines · JEE Main 2025 · Shift-II View Solution on Crucible ↗

12. The distance of the line $\frac{x-2}{2}=\frac{y-6}{3}=\frac{z-3}{4}$ from the point $(1,4,0)$ along the line $\frac{x}{1}=\frac{y-2}{2}=\frac{z+3}{3}$ is :

(A) $\sqrt{17}$
(B) $\sqrt{15}$
(C) $\sqrt{14}$
(D) $\sqrt{13}$

JEE Main 2024 · Shift-I View Solution on Crucible ↗

13. A variable line L passes through the point $(3,5)$ and intersects the positive coordinate axes at the points A and B. The minimum area of the triangle OAB, where O is the origin, is :

(A) 30
(B) 25
(C) 40
(D) 35

Nested Adjoints Involved · JEE Main 2025 · Shift-I View Solution on Crucible ↗

14. Let A be a 3×3 matrix such that $|\text{adj}(\text{adj}(\text{adj}\,A))|=81$. If $S=\left\{n\in\mathbb{Z}:(|\text{adj}(\text{adj}\,A)|)^{\frac{(n-1)^2}{2}}=|A|^{(3n^2-5n-4)}\right\}$, then $\sum_{n\in S}\left|A^{(n^2+n)}\right|$ is equal to

(A) 866
(B) 750
(C) 820
(D) 732

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15. Let S_1 and S_2 be respectively the sets of all $a\in R-\{0\}$ for which the system of linear equations $ax+2ay-3az=1,\ (2a+1)x+(2a+3)y+(a+1)z=2,\ (3a+5)x+(a+5)y+(a+2)z=3$ has unique solution and infinitely many solutions. Then

(A) $n(S_1)=2$ and S_2 is an infinite set
(B) S_1 is an infinite set and $n(S_2)=2$
(C) $S_1=\phi$ and $S_2=R-\{0\}$
(D) $S_1=R-\{0\}$ and $S_2=\phi$

Tricky problems of chord & its properties · JEE Main 2025 · Shift-II View Solution on Crucible ↗

16. Let a circle C pass through the points $(4,2)$ and $(0,2)$, and its centre lie on $3x+2y+2=0$. Then the length of the chord, of the circle C, whose mid-point is $(1,2)$, is :

(A) $\sqrt{3}$
(B) $2\sqrt{2}$
(C) $2\sqrt{3}$
(D) $4\sqrt{2}$

Misc Concepts · JEE Main 2024 · Shift-I View Solution on Crucible ↗

17. The maximum area of a triangle whose one vertex is at $(0,0)$ and the other two vertices lie on the curve $y=-2x^2+54$ at points (x,y) and $(-x,y)$ where $y>0$ is :

(A) 88
(B) 122
(C) 92
(D) 108

Fundamental Definition SP + SQ = 2a · JEE Main 2025 · Shift-II View Solution on Crucible ↗

18. Let the ellipse $3x^2+py^2=4$ pass through the centre C of the circle $x^2+y^2-2x-4y-11=0$ of radius r. Let f_1, f_2 be the focal distances of the point C on the ellipse. Then $6f_1f_2-r$ is equal to

(A) 74
(B) 68
(C) 70
(D) 78

JEE Main 2022 · Shift-II View Solution on Crucible ↗

19. The value of $2\sin 12^\circ - \sin 72^\circ$ is

(A) $\frac{\sqrt{5}(1-\sqrt{3})}{4}$
(B) $\frac{1-\sqrt{5}}{8}$
(C) $\frac{\sqrt{3}(1-\sqrt{5})}{2}$
(D) $\frac{\sqrt{3}(1-\sqrt{5})}{4}$

JEE Main 2024 · Shift-I View Solution on Crucible ↗

20. For $0<\theta<\pi/2$, if the eccentricity of the hyperbola $x^2-y^2\csc^2\theta=5$ is $\sqrt{7}$ times eccentricity of the ellipse $x^2\csc^2\theta+y^2=5$, then the value of θ is:

(A) $\frac{\pi}{6}$
(B) $\frac{5\pi}{12}$
(C) $\frac{\pi}{3}$
(D) $\frac{\pi}{4}$

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21. If $\alpha, -\frac{\pi}{2} < \alpha < \frac{\pi}{2}$ is the solution of $4\cos\theta + 5\sin\theta = 1$, then the value of $\tan\alpha$ is

(A) $\frac{10-\sqrt{10}}{6}$
(B) $\frac{10-\sqrt{10}}{12}$
(C) $\frac{\sqrt{10}-10}{12}$
(D) $\frac{\sqrt{10}-10}{6}$

Finding Sum of Infinite Tan Series · JEE Main 2023 · Shift-II
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22. Let $a_1 = 1, a_2, a_3, a_4, \ldots$ be consecutive natural numbers. Then $\tan^{-1}\left(\frac{1}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{1}{1+a_2a_3}\right) + \ldots + \tan^{-1}\left(\frac{1}{1+a_{2021}a_{2022}}\right)$ is equal to

(A) $\frac{\pi}{4} - \cot^{-1}(2022)$
(B) $\cot^{-1}(2022) - \frac{\pi}{4}$
(C) $\tan^{-1}(2022) - \frac{\pi}{4}$
(D) $\frac{\pi}{4} - \tan^{-1}(2022)$

JEE Main 2023 · Shift-II
View Solution on Crucible ↗

23. The sum of all values of α, for which the points whose position vectors are $\hat{i} - 2\hat{j} + 3\hat{k}, 2\hat{i} - 3\hat{j} + 4\hat{k}, (\alpha+1)\hat{i} + 2\hat{k}$ and $9\hat{i} + (\alpha-8)\hat{j} + 6\hat{k}$ are coplanar, is equal to

(A) -2
(B) 2
(C) 6
(D) 4

Line - Shortest Distance Between Two Lines · JEE Main 2024 · Shift-II
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24. If the shortest distance between the lines $\frac{x-\lambda}{2} = \frac{y-4}{3} = \frac{z-3}{4}$ and $\frac{x-2}{4} = \frac{y-4}{6} = \frac{z-7}{8}$ is $\frac{13}{\sqrt{29}}$, then a value of λ is :

(A) -1
(B) $-\frac{13}{25}$
(C) $\frac{13}{25}$
(D) 1

Centroid and its properties · JEE Main 2023 · Shift-II
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25. Let (α, β) be the centroid of the triangle formed by the lines $15x - y = 82, 6x - 5y = -4$ and $9x + 4y = 17$. Then $\alpha + 2\beta$ and $2\alpha - \beta$ are the roots of the equation

(A) $x^2 - 7x + 12 = 0$
(B) $x^2 - 14x + 48 = 0$
(C) $x^2 - 13x + 42 = 0$
(D) $x^2 - 10x + 25 = 0$

Nested Adjoints Involved · JEE Main 2024 · Shift-II View Solution on Crucible ↗

26. If A is a square matrix of order 3 such that $\det(A) = 3$ and $\det\left(\text{adj}\left(-4\,\text{adj}\left(-3\,\text{adj}\left(3\,\text{adj}\left((2A)^{-1}\right)\right)\right)\right)\right) = 2^m 3^n$, then $m + 2n$ is equal to :

(A) 2 **(B)** 3

(C) 6 **(D)** 4

JEE Main 2025 · Shift-II View Solution on Crucible ↗

27. Let $\alpha, \beta\,(\alpha \neq \beta)$ be the values of m, for which the equations $x + y + z = 1;\ x + 2y + 4z = m$ and $x + 4y + 10z = m^2$ have infinitely many solutions. Then the value of $\sum_{n=1}^{10}(n^\alpha + n^\beta)$ is equal to :

(A) 3080 **(B)** 560

(C) 3410 **(D)** 440

Advanced properties · JEE Main 2024 · Shift-II View Solution on Crucible ↗

28. Let a variable line passing through the centre of the circle $x^2 + y^2 - 16x - 4y = 0$, meet the positive co-ordinate axes at the point A and B. Then the minimum value of $OA + OB$, where O is the origin, is equal to

(A) 12 **(B)** 18

(C) 20 **(D)** 24

Shortest Distance Problems · JEE Main 2024 · Shift-I View Solution on Crucible ↗

29. If the shortest distance of the parabola $y^2 = 4x$ from the centre of the circle $x^2 + y^2 - 4x - 16y + 64 = 0$ is d, then d^2 is equal to :

(A) 16 **(B)** 24

(C) 20 **(D)** 36

Length of LR · JEE Main 2019 · Shift-II View Solution on Crucible ↗

30. In an ellipse, with centre at the origin, if the difference of the lengths of major axis and minor axis is 10 and one of the foci is at $(0, 5\sqrt{3})$, then the length of its latus rectum is:

(A) 6 **(B)** 10

(C) 8 **(D)** 5

Answer Key (Q1 to Q30)

Click any question number below to open its exact step-by-step solution on The Crucible ↗

1: (D)	2: (C)	3: (D)	4: (B)	5: (C)	6: (A)	7: (C)	8: (A)	9: (D)	10: (C)
11: (A)	12: (C)	13: (A)	14: (D)	15: (D)	16: (C)	17: (D)	18: (C)	19: (D)	20: (C)
21: (C)	22: (A)	23: (B)	24: (D)	25: (C)	26: (D)	27: (D)	28: (B)	29: (C)	30: (D)

Sample Solution — Question 1 (Correct Option: D)

Open Full Solution on Crucible ↗

Q1. Let the area of a ΔPQR with vertices $P(5,4), Q(-2,4)$ and $R(a,b)$ be 35square units. If its orthocenter and centroid are $O(2,\frac{14}{5})$ and $C(c,d)$ respectively, then $c+2d$ is equal to

Look at $P(5,4)$ and $Q(-2,4)$: they have the same y-coordinate, so PQ is horizontal. The altitude from R to PQ is then a vertical line through the orthocentre, so R has the same x-coordinate as O.

$R = (2, b)$

• *Use the orthocentre:** O also lies on the altitude from P, so PO is perpendicular to QR.

Slope of $PO = \frac{\frac{14}{5}-4}{2-5} = \frac{2}{5}$

So the slope of QR is $-\frac{5}{2}$.

Slope of $QR = \frac{b-4}{2-(-2)} = \frac{b-4}{4}$

$\frac{b-4}{4} = -\frac{5}{2}$, so $b = -6$

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